Properties of Exponents

This page introduces the properties of exponents one by one in a way designed to help you remember them. If you want a fast reference, all the properties are listed in a table at the end of this page.

PRODUCT OF POWERS PROPERTY

How do you simplify $7^2 \times 7^6$?

If you recall the way exponents are defined, you know that this means:

$(7 \times 7) \times (7 \times 7 \times 7 \times 7 \times 7 \times 7)$

If we remove the parentheses, we have the product of eight 7s, which can be written more simply as:

7^8

This suggests a shortcut: all we need to do is add the exponents!

$7^2 \times 7^6 = 7^{(2+6)} = 7^8$

In general, for all real numbers *a*, *b*, and *c*,

$a^b \times a^c = a^{(b+c)}$

To multiply two powers having the same base, add the exponents.

If you remember only this one and forget the rest, you can use it to figure out most of the other properties.

ZERO EXPONENTS

Many beginning students think it's weird that anything raised to the power of zero is 1. ("It should be 0!") You can use the product of powers property to show why this must be true.

$7^0 \times 7^1 = 7^{(0+1)} = 7^1$

We know $7^1 = 7$. So, this says that $7^0 \times 7 = 7$. What number times 7 equals 7? If we try 0, we have $0 \times 7 = 7$. No good.

In general, for all real numbers a, $a \neq 0$, we have:

$a^0 = 1$

Note that 0^0 is undefined. (Click here to see why.)

NEGATIVE EXPONENTS

You can use the product of powers property to figure this one out also. Suppose you want to know what 5^{-2} is.

$5^{-2} \times 5^{2} = 5^{(-2+2)} = 5^{0}$

We know $5^{2} = 25$, and we know $5^{0} = 1$. So, this says that $5^{-2} \times 25 = 1$. What number times 25 equals 1? That would be its multiplicative inverse, 1/25.

$$5^{-2} = \frac{1}{25}$$

In general, for all real numbers *a* and *b*, where $a \neq 0$, we have:

$$a^{-b} = \frac{1}{a^{b}}$$

QUOTIENT OF POWERS PROPERTY

When you multiply two powers with the same base, you add the exponents. So when you **divide** two powers with the same base, you **subtract** the exponents. In other words, for all real numbers *a*, *b*, and *c*, where $a \neq 0$,

$$\frac{a^{b}}{a^{c}} = a^{b-c}$$

What you're really doing here is cancelling common factors from the numerator and denominator. Example:

$$\frac{9^{5}}{9^{2}} = \frac{9.9.9.\not{9}.\not{9}}{\not{9}.\not{9}} = 9.9.9 = 9^{3}$$

POWER OF A PRODUCT PROPERTY

When you multiply two powers with the same *exponent*, but different bases, things go a little differently.

$3^{2} \times 4^{2} = (3 \times 3) \times (4 \times 4)$

Because of the commutative and associative properties of multiplication, we can rewrite this as

$3^{2} \times 4^{2} = (3 \times 4) \times (3 \times 4) = 12^{2}$

In general, for all real numbers *a*, *b*, and *c* (as long as both *a* and *c* or both *b* and *c* are not zero):

$a^{c} \times b^{c} = (ab)^{c}$

To find the power of a product, either find the power of each factor and then multiply or multiply the factors and raise the product to the power.

POWER OF A QUOTIENT PROPERTY

This is pretty similar to the last one. By canceling common factors, you can see that:

Example 1:

$$\frac{20^3}{4^3} = \frac{5 \cdot \cancel{4} \cdot 5 \cdot \cancel{4} \cdot 5 \cdot \cancel{4}}{\cancel{4} \cdot \cancel{4} \cdot \cancel{4}} = 5^3$$

Example 2:

Simplify

For all real numbers *a*, *b*, and *c* (as long as $b \neq 0$, and *a* and *c* are not both 0):

$$\frac{a^c}{b^c} = \left(\frac{a}{b}\right)^c$$

POWER OF A POWER PROPERTY

The product of powers property can be extended. Suppose you have a number raised to a power, and you multiply the whole expression by itself over and over. This is the same as raising the expression to a power:

$(5^3)^4 = (5^3)(5^3)(5^3)(5^3)$

But the product of powers property tells us that

$(5^3)(5^3)(5^3)(5^3) = 5^{3+3+3+3} = 5^{4(3)} = 5^{12}$

So it is enough to just multiply the powers!

In general, for all real numbers *a*, *b*, and *c*,

$(a^b)^c = a^{bc}$.

To find a power of a power, multiply the exponents.

RATIONAL EXPONENTS

We've covered positive exponents, negative exponents, and zero exponents. But what if you have an exponent which is not an integer? What, for instance, is $9^{1/2}$?

We can fall back again on the product of powers property to find out:

$9^{1/2} \times 9^{1/2} = 9^{(1/2 + 1/2)} = 9^1$

We know $9^1 = 9$, so $9^{1/2} =$. Thus, the exponent ½ works like a square root.

Similarly, $a^{1/3}$ is equivalent to .

and in general

$a^{1/b} = \sqrt[b]{a}$.

and $a^{\frac{c}{b}} = \sqrt[b]{a^c} = \left(\sqrt[b]{a}\right)^c$.

To recap:

Zero Exponent Property	$a^0 = 1, (a \neq 0)$
Negative Exponent Property	$a^{-b} = \frac{1}{a^b} \quad ,a \neq 0$
Product of Powers Property	$a^b \bullet a^c = a^{(b+c)} \quad a \neq 0$
Quotient of Powers Property	$\frac{a^b}{a^c} = a^{b-c} \quad ,a \neq 0$
Power of a Product Property	$a^c \bullet b^c = (ab)^c ,a,b \neq 0$
Power of a Quotient Property	$\frac{a^c}{b^c} = \left(\frac{a}{b}\right)^c \quad a,b \neq 0$
Power of a Power Property	$\left(a^b\right)^c = a^{bc}$
Rational Exponent Property	$a^{\frac{1}{b}} = \sqrt[b]{a} \quad b \neq 0$ $a^{\frac{c}{b}} = \sqrt[b]{a^c} = \left(\sqrt[b]{a}\right)^c$